**The Variation of Helium and Xenon Gas Pressure as a Function of Temperature, Volume and Mole Count**

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**Abstract**

The Forey technique was used to measure He and Xe gas pressures over the ranges: 100-800 K, 0.1-20 moles and 1-200 L. Deviance of observed pressures from Ideal Gas Law predictions modeled using just the van der Waals **a** correction for intermolecular interaction led to: **a**(He)=0.034(atm-L2/mol2) ; a(Xe) =4.25(atm-L2/mol2) in agreement with accepted values. (54 words & no.)

**Introduction**

Using the method of Forey1,  the observed pressure**, P**(atm), of He and Xe over 100-800 K, 0.1-20 mol and 1-200 L are recorded and the results compared against the predictions of the simplified Van der Waals equation2, **1**, where **n** are moles of gas, **T** is temperature in Kelvin, (K), **V** is volume in liters (L), **P** is measured in atmospheres (atm), **R** , is the Ideal Gas Constant taken to be 0.08206 atm-L/K mol, and

**a**(atm L2/mol) is the Van der Waals molecular interaction parameter.

**1** **P = nRT - an2 = Pideal – an2**

 **V V2 V2**

 (99 words & no. )

**Experimental**

The technique and apparatus previously described1 have been employed. Gas sources are: Xe (99.999%, Linde Gas Reagent grade lot #561A-3) and He (99.99%, Air Gas Technologies, Technical grade, lot AG3412). Gases were used as received. Three series of pressure measurements were performed:

1. **n** varied from 0.100 to 8.000 moles at constant **V**=20.000 L and **T**=300.0 K
2. **T** varied from 100.0 to 800.0 K at constant **V**=20.000 L and **n**=1.000 mol
3. **V** varied from 1.000 to 200.0 L at constant **T**=300.0 K and **n**=1.000 mol

The significant figure counts above reflect instrument precision. (100 words & no.)

**Results**

Tables 1-3 below summarize the results of the three series of pressure measurements just described.

**Table 2: Observed P(He) and P(Xe) versus T(K)**

**V=20.000 L and n=1.000 mol**

**Table 1: Observed P(He) and P(Xe) versus n(moles)**

 **V=20.000 L and T=300.0 K**

|  |  |  |
| --- | --- | --- |
| **T( K)** | **P(He) atm** | **P(Xe) atm** |
| 100.0 | 0.4102 | 0.3997 |
| 200.0 | 0.8205 | 0.8099 |
| 300.0 | 1.2308 | 1.2202 |
| 400.0 | 1.6411 | 1.6305 |
| 500.0 | 2.0514 | 2.0408 |
| 600.0 | 2.4617 | 2.4511 |
| 700.0 | 2.8720 | 2.8614 |
| 800.0 | 3.2823 | 3.2717 |

|  |  |  |
| --- | --- | --- |
| **n (moles)** | **P(He) atm** | **P(Xe) atm** |
| 0.100 | 0.1231 | 0.1230 |
| 0.200 | 0.2462 | 0.2458 |
| 0.500 | 0.6154 | 0.6128 |
| 1.000 | 1.2308 | 1.2202 |
| 2.000 | 2.4615 | 2.4193 |
| 5.000 | 6.1524 | 5.8889 |
| 8.000 | 9.8418 | 9.1672 |
| 20.000 | 24.5840 | 20.3680 |

**Table 3: Observed P(He) and P(Xe) versus V(L)**

**T=300 K and n=1.000 mol**

|  |  |  |
| --- | --- | --- |
| **V(L)** | **P(He) atm** | **P(Xe) atm** |
| 1.000 | 24.5840 | 20.3680 |
| 5.000 | 4.9222 | 4.7536 |
| 10.000 | 2.4614 | 2.4193 |
| 20.000 | 1.2308 | 1.2202 |
| 40.000 | 0.6154 | 0.6128 |
| 100.000 | 0.2461 | 0.2458 |
| 200.000 | 0.1230 | 0.1230 |

The deviation, **ΔP,** of the observed pressures versus **n**, **T** and **P** compared to the Ideal Gas prediction from the several tables is computed by **2:**

**2** **ΔP = Pideal – P = nRT - P**

  **V**

Assuming **1**, wherein the volume correction term, **b,** is assumed 0 in the general van der Waals equation, **2**

can be reduced to **3**, a form which can be tested for validity using the data in Tables 1-3.

**3** **ΔP = an2**

 **V2**

Consistent with **3,** Figure 1, which plots **T** versus **ΔP** at constant **V** and **n** from Table 3 reveals a constant, absolute deviance from ideality versus temperature. For He, **ΔP=ΔPT**  is 1.00\*10‑4 atm and for Xe it is 1.07\*10-2 atm.

**50\*ΔPT(He)**

**ΔPT(Xe)**

**P** vs. **n2** and **P** vs. (**1/V2**) plots derived from Table 1 and 2, respectively, yield Figures 2 and 3 respectively. Linear regression analysis of the data from these two plots are summarized in Table 4.

**ΔP(Xe)**

**50\*ΔP(He)**

He

**50\*ΔP(He)**

**ΔP(Xe)**

**Table 4: Linear Regression Coefficients Derived from Figures 2 and 3**

Figure 2: ΔP= mn2 +b Figure 3: ΔP = m’(1/V2) + b-‘

 V=20.000 L, T =300.0K n=1.000 mol, T=300.0 K

Gas m b r2 m’ b’ r2

He 8.50\*10‑5 -5.10\*10-5 0.99999 3.39\*10-2  5.21\*10‑5  0.99999

Xe 1.06\*10-2 -2.44\*10‑6 1.00000 4.25 1.24\*10‑6 1.00000

  **atm/L2 mol2 atm atm L2/mol2 atm**

Since the uncertainty in pressure measurement is assumed to be the precision limit of the Forey apparatus, ±1\*10‑4 atm, the intercept values above can be approximated to be zero, allowing direct estimation of the **a** from **3** and the slopes, **m** and **m’**. From the fit of Figure 2:

**4a** **ΔP = an2 = a\*n2 = mn2**

 **V2  202**

Similarly, from the fit of Figure 3:

**4b** Δ**P = an2 = a = m’**

 **V2  V2 V2**

Table 5 summarizes the computed and accepted3 values of **a** for He and Xe derived from **4a** and **4b,** along with the computed versus observed values of the deviance, **ΔPT**  shown in Figure 1 assuming the computed **a**  with **V**=20.000 L and **n**=1.000 mol in **3.**

**Table 5: Observed vs. Expected van der Waals Interaction Constant a and ΔP1 computed from** **4a, 4b** and **3**

**gas a(from 4a) a(from 4b) a(accepted)3 ΔPT (computed) ΔPT(observed)**

He 0.034 0.034 0.034 8.50\*10-5  1.00\*10-4

Xe 4.25 4.25 4.25 1.06\*10-2 1.07\*10-2

 units in (atm L2 mol-2) units in atm (485 words & no.)

**Discussion**

The van der Waals interaction term a is an empirically derived macroscopic constant that reflects the degree to which a specific gas’s intermolecular attraction to itself decreases the velocity of its’ gas molecules as it approaches the walls of the volume they are contained in. Such attraction has the effect of lowering the observed pressure from that predicted by the Ideal Gas Law where no velocity damping interactions are assumed. In an intuitive and strictly qualitative sense, a is akin to **k** in Coulomb’s equation:

**5**  **F= k q1q­2**

**r2**

 Like **k**, which quantitatively characterizes the absolute magnitude of the interaction of two charges **q1** and **q2** at a distance **r**, a scales the attractive interaction between two hypothetic gas densities **d1**=**n1**/**V** and **d2=n2/V** as measured by the increase in pressure, **ΔP**, a gas would attain if no such velocity-dampening attraction occurred:

**6** **ΔP** = a d1d2 = **a n1 n2**

  **V2**

Since we are interested in the interaction between identical densities of a single gas, **d**= **n/V**,

**6** simplifies to the familiar form below:

**3 ΔP = an2**

 **V2**

The foregoing clearly predicts that increasing non-ideality occurs as gas density is increased. For example, at a common gas density of 0.005 mol/L (see Table 1), He and Xe exhibit no measurably significant deviance from ideality. But, at a density of 1 mol/L, He deviates by 0.14% and Xe deviates by 17.3%. Similar to the effect of increasing charge density in Coulomb’s law, increasing gas density correlates with increased attraction between gas molecules. Higher density necessarily lowers the distance between gas molecules and thus increases the magnitude of their interaction in a fashion analogous to Coulomb’s Law.

 Less obviously, the presence of a fixed, finite attractive interaction at constant **n/** **V** implied by **3** also reveals how deviance from ideal behavior depends on temperature. Using the data of Table 1, the % deviance of P from ideality of both He and Xe as a function of **T**, are shown in Figure 4. As expected, at and above room temperature molecular energies are evidently high enough so that intermolecular interactions, which are fixed in magnitude, produce less than 1% deviations. Deviance becomes noticeable, however, as temperatures fall below 250 K where the kinetic energy to overcome the fixed attractions is much lower.

**50\*He %**

**Xe %**

The present work confirms previous estimates3 of a . The obviously wide difference between He and Xe,

whose a differ by a factor of a(Xe)/a(He)= 125 correlates well with molecular scale measurements of polarizability and binding energy for the two gases as seen Table 6.

**Table 6: Comparison of Derived Macroscopic Interaction Constants a for He and Xe**

 **versus Selected Molecular Interaction Measurements**

**gas a(atm L2/mol2) α(polarizability, AU)4 Binding energy Pd(100) face, MeV4**

He 0.034 1.322 10

Xe 4.25 27.97 320-360

ratio 125 21 32-36 (463 words & no. )

**References**

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