**Chem 6854: Physical Chemistry**

**Homework Assignment #2**

 **Solving Homogenous, 2nd order, constant coefficient Diff. Eq.;**

**Using Diff. Eq to find solutions to classical physics problems**

 35 points total

Show all work !

 Due Wed 10 February

**Answers:**

2.1 Text problems 2-1a , c and e (3 pts each) page 54 9 pts

2.1a. Assuming the solution of the form erx=> for y” -4y’ +3y=0 using the recipe derived in class:

*a=1, b= -4, c= 3=> r= [-b±(b2-4ac)1/2]/2a = [+4 ± ((-4)2 -4\*1\*3)1/2]/2= [4 ±(16-12)1/2]/2= [4±2]/2 = 3, 1*

*∴y =c1ex + c2e3x*

*2.1c 0\*y” + y’ + 3y=0 => a = 0 which means we can’t use quadratic, so we simply solve the simpler equation:*

 *dy/dx = -3y*

Assume as usual that y= erx => d(erx) = rerx = -3erx => r= -3=> general solution of form:

∴ y = c1e-3x

2.1e. y” -3y’ +2y = 0 . Solve as per 2.1a with a= 1, b=-3, c=2=> r= [+3±((-3)2 -4\*1\*2)1/2]/2 = [3±1]/2= 2, 1

*∴y =c1ex + c2e2x*

2.2 Text problems 2-2a, b (3 pts each) page 55 6 pts

*2.2a solve as per 2.1, a= 1, b=0, c=-4 => r= ±(-4\*1\*(-4))1/2/2 = ±4/2 = ±2*

*∴general solution for y =c1e2x + c2e-2x*

*Applying initial conditions to find c1 and c2:*

*y(0) =2 = c1 +c2*

*dy(0)/dx= 4= 2c1 -2c­2*

 *2= c1-c2*

*Solving for c1 we add the two equations and find 4=2c1=> c1=2*

*Substitute into one of the equations above=>c2=0*

*∴ specific solution satisfying initial conditions: y(x) = 2e2x*

*2.2b solve as per 2.1, a= 1, b=-5, c=6 => r= 5±((-5)2 -4\*1\*(6))1/2/2 =[ 5 ± 1]/2 = 3,2*

*∴general solution y =c1e2x + c2e3x*

*Applying initial conditions to find c1 and c2:*

*y(0)= -1 =c1 + c2*

*dy(0)/dx= 0 = 2c1 + 3c2*

* c1=- 3c2 /2

-1= -3c2/2 + c2 = -½c2=> c2 = 2 so c1 = -3

 *∴specific solution satisfying initial conditions : y =-3e2x + 2e3x*

2.3. Text problem 2-5 (3 pts) page 55 4 pts

***Showing A\*sin(ωt + φ) is the same as c1 cos ωt + c2 sin ωt***

*Using the identities provided ωt= α, φ=β: x(t) = A\* sin(ωt + φ) = A\*[sin (ωt)cos φ + cos(ωt)sin φ]*

*Observing that φ = constant => sin φ and cos φ are also constant so we distribute A across the sum just derived and set:*

 *c1 = A\*cos φ, c2= A\*sin φ.*

*∴A\*[sin (ωt)cos φ + cos(ωt)sin φ] = A\*cos φ sin (ωt) + A sin φ cos(ωt) = c1 cos ωt + c2 sin ωt*

***Showing B\*cos(ωt + ψ) is the same as c1 cos ωt + c2 sin ωt***

*Using the identities provided ωt= α, φ=β: x(t) = B\* cos(ωt + ψ) = B\*[cos (ωt)cos ψ - sin(ωt)sin ψ]*

*Observing thatψ= constant => sin ψ and cos ψ are also constant so we distribute B across the sum just derived and set:*

 *c1 =- B\*sin ψ,* c2 = B\*cos ψ

*∴ B\*[cos (ωt)cos ψ - sin(ωt)sin ψ]= -B\*sinψ sin (ωt) + B\*cos ψ\*cos (ωt) = c1 cos ωt + c2 sin ωt*

2.4 Text problem 2-6 page 55 4 pts

c1e-xe3ix +c2e-x e-3ix =c1e-x(cos 3x + isin 3x) +c2e-x(cos(-3x) + i sin(-3x))

 = c1e-x(cos 3x + isin 3x) +c2e-x(cos(3x) – isin 3x)

group terms: (c1+c2)e-x cos 3x + (c1-c2)e-x cos(3x) =e-x((c1+c2)cos 3x + (c1-c2)cos(3x))

Redefine constants: (c1+c2)=c3, (c1-c2)=c4 and substitute in last equation re-capitulates desired form:

e-x(c3 cos 3x + c4 cos(3x))

1. r1, r2= (½ )(-2 ± (4-4\*1\*2)1/2 =-1 ± i=>y=c1 e(-1+i)x +c2e (-1-i)x = e-x(c1eix +c2e-ix)=**ex(C3cos(x) + C4 sin(x))**
2. r1,r2 = ½ ( +6 ±(36-4\*25)1/2)= ½ (6 ± 8i)=3± 4i=>y= e3x(c1e4ix +c2e-4ix)= **e3x(C3cos(4x) + C4 sin(4x))**
3. r1,r2 = ½ ( -2β ±(4β2-4\*(β2 + ω2))1/2)= ½ ( -2β ±(-4 ω2))1/2)=-β± iω=> **y=e-βx(C3cos(ωx) + C4 sin(ωx))**
4. r1,r2 = ½ (-4 ±(16-4\*5)1/2= ½ (-4± 2i)=-2±i => y=e-2x(C3cos(x) + C4 sin(x))

Since y(0)= 1=> C3=1; since dy/dx=-3 at x=0=> dy/dx= -2e-2x(cos(x)+ C4sin(x)) +e\_2x(-sin(x)+C4cos(x))

And at x=0 dy/dx= -2(1) +1\*(C4)=-3=>C4=-1=> **y= e-2x(cos(x) –sin(x)**

2.5 Text problem 2-7. For simplicity, use capital X= x-xo rather than the ξ (Greek `chisee’, a rotten symbol to write or type). Note also, that to find C in the general solution X= Csin((k/m)1/2t, you use the initial conditions for velocity at t=0 = vo =dX/dt at t=0.

*This was done in class and should be in your notes*

Instead of `interpreting and discussing’ the final derived equation as stated in text provide an Excel plot of X(t) vs t for the specific case of k=5, , m=1, vo =20 from t=0 to 3π for the derived equation

 X(t) = vo(m/k)1/2 sin ([k/m]1/2t) 6 pts

2.6 Problem 2.18 (the pendulum at low displacement θ where sin θ~ θ) 6 pts

*The component of Mg that acts to oppose the pendulum Fpend can be deduced so*

*(Use of fact that perpendiculars to arc form parallel lines and equal angle relations*

*arise)*

 **θ**

Fmg = Mg sin θ

Since the Fpend is opposite this:

Fpend = -Fmg = -Mg sin θ

 **θ**

 **Mg Component along s =Fmg**

 **θ**

 **θ**

 **Mg component perpendicular to s**

 **Mg**

***The final D.E. is thus:***

***ML d2θ(t) = -Mg sin θ(t)***

 ***dt2***

***In general, this turns out to be difficult to solve since θ(t) shows up inside as an argument to a sine function.***

***We can approximate sin θ(t) ~ θ(t) as θ(t) becomes small (e.g. we don’t swing pendulum back to far ) as suggested by the text. Then:***

 ***L d2θ(t) ~ -g θ(t)***

 ***dt2***

* *D.E is :*

***θ” + g/L θ = 0***

***a=1, b=0, c= g/L => roots = ± (-4g/L)1/2 = ±(g/L)1/2***

 ***2***

***General solution at low displacement : θ(t) = C1 sin √(g/L)t + C2cos √ g/L t***

***If we assume θ(0) = 0 as an initial condition, then θ(t) = C1 sin √(g/L)t***

*If we assume θ(0) = θo as initial condition, then* ***θ(t) = C2 cos √(g/L)t***

*In general,* ***cos (x)*** *repeats when x=2π when* ***√(g/L)t = 2π. The equivalent t when repetition commences is the `time constant’, τ (tau) which we derive by rearranging √(g/L)t = 2π to isolate t satisfying the equation:***

***Time constant τ = 2π√(L/g) =>***

***θ(t) = C2 cos √(g/L)t =***  *C2 cos(2πt/τ)*