**Supplement 5: Chemistry 6854**

**Physical Chemistry Alfred State College**

**Classical Description of Single Rotating Mass and Extensions to Two Body Rotator**

v m

**5.1 Angular Momentum L of Single Rotating Particle, m: r**

**ω**

The angular momentum of a particle m fixed at a distance r from

a point and rotating with angular speed ω is a vector quantity

(has both magnitude and direction) .It’s computed via a `cross product’,

which is compactly written:

**L = r x p**

where **r** and **p** are vectors and **x** means cross product. The magnitude of **L** is L:

L =r\*p\*sin θ = numeric magnitude of **L**.

The direction of the **L** vector is ascertained using the right hand rule which `crosses **r** into **p** by curving your right hand’s fingers from **r to p** and noting the direction the thumb.For a single rotating particle:

L **=** r\*mv sin 90 = rmv

Since v= ωr, we can re-write L=rmv =rm (ωr) = mr2ω .

In analogy with linear momentum, p, which has the well-known form:

p=mv **linear momentum**

L can be written in terms of its angular speed ωas shown below in **1**:

**1** L =Iω **angular momentum**

I= mr2 is called the moment of inertia of the system.

**5.2. Angular (Rotational) Kinetic Energy, Erot of a Single Mass**

Recall that linear, kinetic energy has the form:

Elin = ½ p2/m  **linear kinetic energy**

Angular kinetic energy has an analogous form, **2:**

**2** Erot = L2/2I **1-body angular kinetic energy**

Since L= Iω, we can substitute into 2, which yields a form analogous Elin= ½ mv2, equation **3**:

**3** Erot = ½ Iω**2 alternate form of 1-body angular kinetic**

**energy**

**5.3 Extension to Two Body System**

Recall from our analysis of spring motion that the use of a reduced mass μ allows us to view the motion of two masses, m1 and m2 as if they were just a single reduced mass In simple terms, this is like assuming we are observing μ while assuming that our perch on m1 is not moving during vibration. The same vantage point and simplification is applicable for rotation.

That is, we assume we are observing a mass μ rotating around us wherein we act as the motionless center point, e.g., we are observing a one body system as described in 5.1 and 5.2 above, where the rotating mass has the magnitude:

μ = m1m2

m1+ m2

m1 r m2

c.m.

Thus, for a two body rotation, we can substitute m in with μ in all the equations in 5.1. and 5.2 above. The substitutions are summarized below.

**One body Two body**

I mr2 μr2

L Iω =mr2ω Iω=μr2ω

Erot ½ Iω2 = ½ mr2ω2 ½ Iω2 = ½ μr2ω2